the value

$$\frac{I_B^{**}}{I_s} = \frac{2(d/a)(w/a)[2 - (w/a)][\frac{1}{3}(d/a)^2 + (1 + k^2)/4]}{(h/a)[\frac{1}{12}(h/a)^2 + \frac{1}{4}]} \approx 0.097 \tag{14}$$

and an upper limit for the moment of inertia.

The effective moment of inertia ratio is therefore

$$I_e/I_s = (I_e/I_s)_{swb} + I_B^{**}/I_s + (I_e/I_s)_{bd} \approx 0.346 \tag{15}$$

For a container with five baffles, the effect of the two additional baffles has to be added by multiplying the effective moment ratio of one baffle by $[1 + 4(d/a)^2]$. It is therefore

$$\frac{I_e^{(5)}}{I_r} = \left(\frac{I_e}{I_r}\right)_{swb} + \left(\frac{I_e}{I_r}\right)_{b_1} + 2\left(\frac{I_e}{I_r}\right)_{bd} + 2\left(\frac{I_e}{I_r}\right)_{b_2d} \tag{16}$$

which yields here

$$I_e^{(5)}/I_r = 0.21 + 0.033 + 0.077 + 2(0.054) = 0.43 \tag{17}$$

Again this value is smaller than the measured one. Considering the liquid completely trapped between the baffles would yield a value

$$I_e^{(5)}/I_r = (I_e/I_r)_{swb}(I_{r_{r=b}}/I_{r_{r=a}}) + I_B^{*}/I_r \sim 0.51 \tag{18}$$

where the value for the trapped liquid has been obtained from

$$\frac{I_B^*}{I_r} = \frac{(w/a)[\frac{1}{12}(h/a)^2 + (1 + k^2)/4][2 - (w/a)]}{[\frac{1}{12}(h/a)^2 + \frac{1}{4}]} \approx 0.38. \tag{19}$$

The following table summarizes the results of theoretical approximations and the experiments (n is number of baffles).

References

[1] Dodge, F. T. and Kana, D. D., "Moment of inertia and damping of liquids in baffled cylindrical tanks," J. Spacecraft Rockets **3**, 153–155 (1966).

[2] Bauer, H. F., "Fluidoscillations in the container of a space vehicle and their influence upon stability," NASA Marshall Space Flight Center, TR-R-187 (February 1964).

[3] Bauer, H. F., "Approximate effect of ring stiffener on the pressure distribution in an oscillating cylindrical tank partially filled with a liquid," Army Ballistic Missile Agency Rept. DA-M-114 (September 1957).

[4] Kirchhoff, G., "Zur Theorie freier Flüssigkeitsstrahlen," Crelle J. Math. **70**, 289–298 (1869).

[5] Prandtl, L., *Führer durch die Strömungslehre* (Friedr. Vieweg und Sohn, Braunschweig, Germany, 1949), pp. 170–173.

Reply by Authors to Helmut F. Bauer

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THE authors welcome Professor Bauer's comments, which should be added to his already long list of contributions in the study of the dynamic behavior of liquids in moving tanks.

Before making a detailed reply, we would like to correct two slight errata in our Note (1). First, the heading for Table 1 should read $n = 0, 1, 3, 5$ instead of $n = 0, 1, 2, 3$ as printed. Second, in the calculations presented below Eq. (5) of the Note, the numerical values should be corrected to read $I_{\text{liq}} = 167$ kg − cm², $I_{\text{rig}} = 331$ kg-cm², and $I_{\text{liq}}/I_{\text{rig}} \approx 0.51$. These corrections indicate that the approximate theory and experiments agree even better than originally implied.

Dr. Bauer has given a very plausible method of calculating the moment of inertia of the liquid in a baffled tank. The arguments he uses [which lead to Eq. (1)] for calculating the moment of the inertia when the liquid near the baffles can be considered as rigid are quite similar to the ones we presented in our Note for the same case. His further theoretical work, which permits the calculation of moment of inertia for tanks in which the baffles are not closely spaced, is indeed valuable and will be of considerable help to missile designers. We are especially glad to see the close correlation between his theory and our test results; this gives us further confidence that both his theory and our experiments are essentially correct.

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Reference

[1] Dodge, F. T. and Kana, D. D., "Moment of inertia and damping of liquids in baffled cylindrical tanks," J. Spacecraft Rockets **3**, 153–155 (1966).

Comments on "Approximate Re-Entry Velocity and Heating Equations"

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GLOVER[1] developed approximate equations developed for an atmosphere in hydrostatic equilibrium describing the time of flight and the total convective heat input at the stagnation point. It is the intent of this comment to cast these equations in a slightly different form which will allow their evaluation from standard mathematical tables. The particular equations derived here will assume an exponential atmosphere; however, one will recognize the equations to be of the same form as for any atmosphere in hydrostatic equilibrium. The nomenclature used herein is consistent with standard re-entry literature.

The velocity history may be approximated by[2]:

$$V = V_E e^{-\alpha e^{-\beta y}} \tag{1}$$

where α is defined by

$$\alpha \equiv \rho_0 C_D A / 2\beta m \sin\phi \tag{2}$$

The equation governing the time of flight becomes

$$t - t_0 = \frac{1}{\beta V_E \sin\phi} \int_{x_0}^{x} \frac{e^x}{x}\, dx \tag{3}$$

where $x = \alpha e^{-\beta y}$. The integral in Eq. (3) may be evaluated in terms of the exponential integral tabulated in Ref. 3:

$$\int_{x_0}^{x} \frac{e^x}{x}\, dx = \int_{-\infty}^{x} \frac{e^x}{x}\, dx - \int_{-\infty}^{x_0} \frac{e^x}{x}\, dx = Ei(x) - Ei(x_0) \tag{4}$$

Making use of a series expansion for the exponential integral in Ref. 4, we may write

$$Ei(x_0) = \ln x_0 + \gamma + x_0 + (x_0^2/2\cdot 2!) + \ldots \tag{5}$$

where γ is Mascheroni's or Euler's constant. Assuming that

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